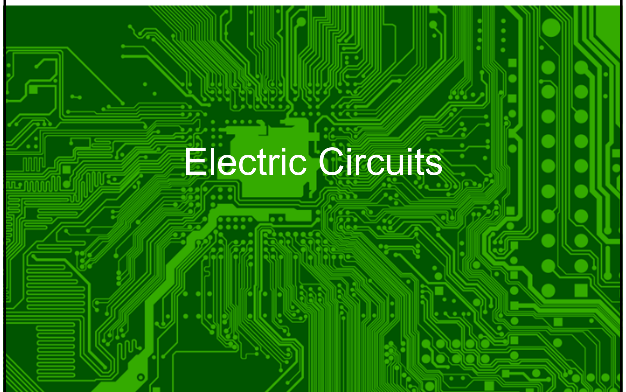
Electric Circuits



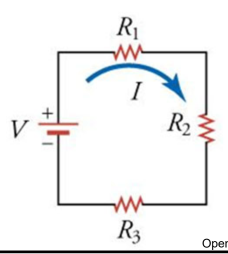
Credit: tagawa (Openclipart)

- The current is the same everywhere in a simple electric circuit.
- The voltage drops as you go through a resistor (or component).
  - Energy is used by the resistor (or component).
- The voltage drop across the resistor can be calculated with Ohm's law.

$$V_{resistor} = IR$$

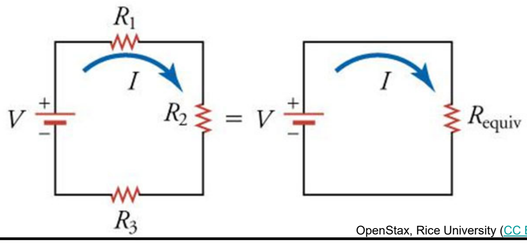
Resistors in Series

- Components connected in series are connected one after the other in the same branch of a circuit.



OpenStax, Rice University (CC BY 4.0)

- We can calculate an equivalent resistance to the resistors in this circuit.
  - An equivalent resistor is a resistor that has the same resistance as the combined resistance of a set of other resistors.
  - $V_{battery} = IR_{equiv}$



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- In a series circuit, the same current flows through all the components.
  - There is only one path.
- The voltage drop across each resistor is $V = IR$.
- The sum of these voltages must equal the output of the battery.

$$V_{battery} = V_1 + V_2 + V_3$$

$$V_{battery} = IR_1 + IR_2 + IR_3$$

$$V_{battery} = I(R_1 + R_2 + R_3)$$

$$\frac{V_{battery}}{I} = R_1 + R_2 + R_3$$

but $$R_{equiv} = \frac{V_{battery}}{I}$$

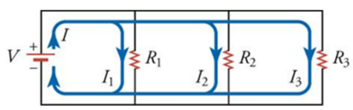
so

$$\boxed{R_{equiv} = R_1 + R_2 + R_3}$$

- A circuit with resistors in series is known as a **voltage divider**.
  - The voltage is divided among the resistors.

Resistors in Parallel

- Components are in parallel when both ends of each component are connected directly together.
  - There are multiple ways for the current to travel.



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- In a parallel circuit, the voltage drop across each component is the same.
  - They are connected (to the same wire) on both sides of the component.
- The current through each component may be different as the resistance may be different.
- The voltage across each resistor is

$$V = I_1R_1 = I_2R_2 = I_3R_3$$

- Rearranging these equations gives

$I_1 = \frac{V}{R_1} \quad I_2 = \frac{V}{R_2} \quad I_3 = \frac{V}{R_3}$

- Charge is conserved. Therefore, the sum of the individual currents is the total current in the circuit, I.

$$I = I_1 + I_2 + I_3$$

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\frac{I}{V} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

but

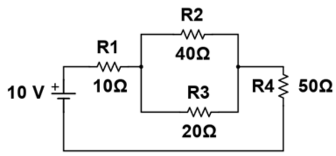
$$\frac{1}{R_{equiv}} = \frac{I}{V}$$

so

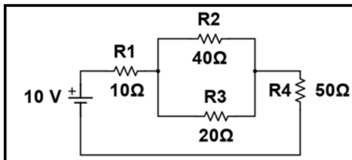
$$\boxed{\frac{1}{R_{equiv}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

- A circuit with resistors in parallel is known as a **current divider**.
  - The current is divided among the resistors.

Example



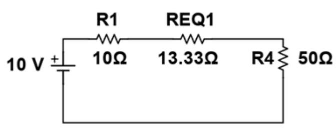
Calculate the equivalent resistance of the circuit, the voltage drop across each resistor, and the current through each resistor.



R2 and R3 are in parallel.

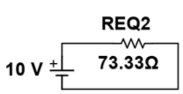
$$\frac{1}{\text{REQ1}} = \frac{1}{40} + \frac{1}{20}$$

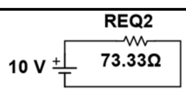
$$\text{REQ1} = 13.33\Omega$$



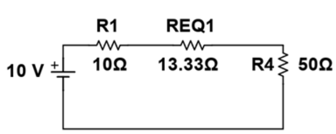
R1, REQ1, and R4 are in series.

$$\text{REQ2} = 10 + 13.33 + 50$$
$$\text{REQ2} = 73.33\Omega$$





$$I = \frac{V}{R} = \frac{10}{73.33} = 0.136 \text{ A}$$



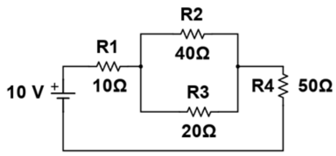
R1, REQ1, and R4 are in series. Therefore, all have the same current.

$$I_{\text{R1}} = I_{\text{REQ1}} = I_{\text{R4}} = 0.136 \text{ A}$$

$$V_{\text{R1}} = (0.136)(10) = 1.36 \text{ V}$$

$$V_{\text{REQ1}} = (0.136)(13.33) = 1.81 \text{ V}$$

$$V_{\text{R4}} = (0.136)(50) = 6.8 \text{ V}$$



R2 and R3 are in parallel. Therefore, the voltage drop is the same across both resistors.

$$V_2 = V_3 = 1.81 \text{ V}$$

$$I_{\text{R2}} = \frac{1.81}{40} = 0.045 \text{ A}$$

$$I_{\text{R3}} = \frac{1.81}{20} = 0.091 \text{ A}$$

Power

- Electric power is the rate at which electric energy is transferred in a circuit.

$$P = IV$$

  - The power could be converted to mechanical energy, for example in a motor.
  - The power could also be dissipated as heat from the resistance in the circuit.

- We can use Ohm's law to substitute for either voltage or current.

$$P = I^2 R$$

$$P = \frac{V^2}{R}$$